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REMARKS ON AIRPLANE STRUTS AND GIRDERS UNDER
COMPRESSIVE AND BENDING STRESSES.

INDEX VALUES.

By Herbert Wagner

From Zeitschrift für Flugtechnik und Motorluftschiffahrt
June 14, 1928

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S t r u t s

Index value of a strut.-- We shall consider the behavior of straight, centrally loaded compression struts. Widely varying phenomena appear in the failure of such a strut. Struts which break clean, according to Euler, are poor from the standpoint of strength, as will be explained later. In most cases the yield point is reached or exceeded, at least in certain places, before failure occurs. Buckling and torsional phenomena also appear. We now have no method which enables the computation of the buckling load of a strut with the degree of accuracy requisite in airplane construction and with due regard to all these complex phenomena. Even if we had such a method, it would be too complicated. For determining the dimensions of a strut, we are therefore usually compelled to resort to the evaluation of experiments.

The basis for this evaluation is supplied by the law of similarity of the strength of materials. This law says that, with two geometrically similar struts, made of exactly the
*"Einige Bemerkungen über Knickstäbe und Biegungsträger. Der Kennwert." From Zeitschrift für Flugtechnik und Motorluftschiffahrt, June 14, 1928, pp. 241-247.

same material and similarly loaded from the geometric standpoint, all deformations will be geometrically similar and all geometrically corresponding points will be subjected to like stresses, provided the external loads are proportional to the square of the linear dimensions of the struts. This law applies even when the yield point is exceeded. Hence doubling the linear dimensions (and therefore the length l) of the strut increases the buckling load P fourfold; tripling the linear dimensions increases P ninefold, etc. It is obvious that the geometrically similar enlargement of a strut does not alter the value of $\sqrt{P/l}$, which we will call the index value K of the strut. That is $K = \sqrt{P/l}$.

If, for example, a strut has a circular cross section, and if its length l_1 is such that the yield point of the material is barely reached at the buckling load, then the material is equally well utilized in all geometrically similar struts, i.e., with those having the same index value. If we consider another strut which must withstand the same buckling load, $P_2 = P_1$, as the original strut, but which is supposed to have twice its length $l_2 = 2l_1$, we must then, if we wish to give the new strut also a circular cross section, increase the inertia moment of the cross-sectional area and also increase this area itself to correspond with Euler's formula. This, however, diminishes the buckling stress. Therefore, the new strut (as well as all struts geometrically

similar to it) is not so well utilized, and its index value is smaller $K_2 = K_1/3$.

In order to utilize the material of the new strut better, we can, for example, correspondingly increase the moment of inertia by using a circular cross section (tube) without increasing the cross-sectional area. Then, even with this strut, the yield point is nearly reached with a smaller index value, and the material is therefore well utilized. If the index value of a strut is still smaller, the walls of the tube must be made still thinner in proportion to its diameter, i.e., the cross-sectional area of the walls of the tube must be made smaller. Despite this cross-sectional reduction, the buckling stress diminishes with diminishing index value, since buckling phenomena now begin to appear. Hence it would be disadvantageous to use this greatly reduced cross section for a strut with high index value..

The modulus of elasticity plays hardly any role with very short, highly stressed struts, hence with very large index values. Everything then depends on the breaking strength of the material. Very long, weakly stressed struts, hence with small index values, require, however, a material with a relatively high modulus of elasticity, i.e., a material of high resistance to buckling stresses.

Hence the index value $\sqrt{P/l}$ determines the cross-sectional form of the strut and also the material. The smaller

the index value, the more the cross-sectional form is reduced (i.e., the thinner the walls), the more the buckling phenomena appear, and the smaller the attainable buckling stress with the best cross-sectional form.

The index value depends simply on the external conditions to which the strut is subjected, namely, the axial load and the length of the strut. Both these quantities are determined by the design. The index value is independent of the quantities (material, cross-sectional form and area, buckling stress and weight) sought in the design. It is the designer's task to find the best cross-sectional form and the best material for the given index value.

The index value is not nondimensional. Neither is it possible to find for it an equivalent, nondimensional value. If the force is expressed in kilograms and the length in centimeters, the index values of an airplane structure vary between 0.3 and $3 \text{ kg}^{1/2} \text{ cm}^{-1}$. Any function of the index value would serve the same purpose as the index value $\sqrt{P/l}$. The above value is chosen because the Euler curves then appear as straight lines in the diagrams.

Index value and light construction.-- If the index values occurring in airplane construction are compared with those of high iron structures, it is found that no great difference exists and that geometrically similar cross sections could therefore be used in both cases. The fact, nevertheless, that

in airplane construction, buckling introduces new problems and leads to other constructive solutions is due, aside from the greater accuracy with which the buckling load must be known, to the actual size of the structural parts. In high iron construction with small index values, the cross section is resolved into single struts. This method has also been adopted in airship construction, due to its exceedingly small index values. In airplane construction, with its small dimensions, such a detailed resolution of the strut cross section would be too complicated and costly. The individual components would also be too easily broken. We therefore proceed simply from the thickness of the metal sheets. Buckling phenomena are thus produced, which are avoided by corrugations and angles in the cross-sectional form.

Airplane construction is also distinguished from bridge building by the sheet-metal surface covering. Buckling phenomena also play an important role in the application of this sheet metal. Corresponding to the importance of struts and sheet-metal walls or coverings in metal-airplane construction (together constituting perhaps 90% of the weight of the cell), the art of light-metal airplane construction might be defined as the avoidance of buckling in the struts and sheet-metal walls.

Use of the index value for the dimensioning and comparison of struts.— Even if one has experimented and plotted in the usual way the buckling stress σ for each profile against the slenderness ratio l/i , he thus obtains no information as to which strut is the lighter for a given length and a given axial load.

The following method is preferable. The index values K are entered on the abscissas and the axial or buckling stresses σ on the ordinates. The points obtained in a series of experiments with struts of like cross-sectional form,* but of different lengths, are plotted. Figure 2 represents the experimental results with three cross-sectional forms, all belonging to the same kind of profiles.**

The Euler curve for each cross-sectional form is the straight line passing through the origin. For comparison, Figure 1 shows the usual representation, for the same profiles, of the axial stress σ plotted against the slenderness ratio l/i . If it is desired to extend the results to intermediate wall thicknesses through interpolation, then both diagrams should be plotted, in order to have a control for the estimation.

Example of dimensioning.— $P = 4000 \text{ kg (8818 lb.)}$;

$l = 90 \text{ cm (35.43 in.)}$; $K = \sqrt{P/l} = 0.705$. If the kind of pro-

*"Like cross-sectional form" means a geometrically similar form, while "like profile kind" means similar external dimensions, but varying wall-thickness ratios.

**The curves in this and subsequent figures make no claim to mathematical accuracy.

file shown in Figure 2 has been decided upon for constructional reasons (e.g., suitable combination possibilities), it then follows from this figure that, for this index value, profiles with $a/s = 50$ down to about 35 withstand the greatest axial loads, and in fact $\sigma = 1700 \text{ kg/cm}^2$ (24180 lb./sq.in.). From this the cross-sectional area is found to be $F = P/\sigma = 2.35 \text{ cm}^2$ (.364 sq.in.). We now have to take from our profile table any profile which has a value $a/s = 35-50$ and as near as possible to the given area F . The single profile with normal dimensions, which comes into the question, is probably the profile with $a = 50 \text{ mm}$ (1.97 in.), $s = 1.2 \text{ mm}$ (.047 in.), and $F = 2.55 \text{ cm}^2$ (.395 sq.in.). In contrast with the usual method, it is seen that the process of computation is perfectly definite and very simple.

The dash-and-dot enveloping curve shows the maximum buckling stress of this kind of profile. It is apparent that the thin-walled profiles are superior at small index values and the thick-walled profiles at large index values. It is also manifest that the most favorable cross-sectional form always belongs to a profile which does not buckle according to Euler, but, in buckling, bulges or is stressed beyond the yield point.

Of course the curves for entirely different kinds of profiles can also be plotted in Figure 2, and then the strength characteristics of these profiles can be directly com-

pared. If struts of different materials are compared, then the value σ/γ can be plotted instead of σ , or, as happens in Figure 3, two different scales can be chosen for the values of σ corresponding to the different specific gravities γ . With the aid of such a diagram more accurate weight estimates can also be very quickly made, since the buckling stress can thus be determined without first finding the dimensions.

Buckling resistance of struts made of various materials.--

Comparisons will now be made between the weights of struts of like length and loading, but of different materials. The ratio of these weights depends on the index value and on the kind of profile chosen. A universally applicable proportionality factor cannot be given. Certain limiting cases can be considered, however.

1. Very great index value .-- In this case the yield point $\sigma_{0.2}$ is important in the first approximation. The lightness of the strut then depends on the value $\sigma_{0.2}/\gamma$. This value, as likewise the comparative values introduced farther on, is called the "coefficient of merit" of the material (Cf. B. A. Schroeder, in Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1928, p.105) $G_m = \sigma_{0.2}/\gamma$.

2. Geometrically similar cross sections.-- If two struts are made in geometrically similar cross sections from different materials, and if yield points and buckling phenomena play

no part (which can hardly happen in practice), we then obtain from Euler's formula for this case the coefficient of merit of the material $G_m = E^{1/2}/\gamma$ in which E is the modulus of elasticity.

3. Buckling and bulging of profiles with curved surfaces (e.g., round and streamlined tubes).— As already mentioned, struts in which the yield point is not reached, are always so made that they bulge in buckling. Hence it necessarily follows that two struts of different material, even when they have the same kind of profile, are not made with geometrically similar cross sections, but the thickness of their walls is adapted to the danger of bulging. If the profiles are curvilinear, then the coefficient of merit of the material is determined by the following process, which is applied to round tubes for the sake of simplicity.

With a small wall thickness s , the inertia moment $J = \frac{1}{2} F r^2$ for a tube of radius r and cross-sectional area F . Hence, according to Euler, the buckling moment is

$$P = \pi^2 \frac{E F r^2}{2 l^2} \quad (1)$$

The stress σ , at which bulging occurs in a compressively stressed metal sheet, is determined from the equation $\sigma = k E \frac{s}{r}$ in which k is a constant (Cf. Rudolf Meyer, "Die Knickfestigkeit"). In the most favorable case, buckling and bulging occur nearly simultaneously and

$$P = F \sigma = F k E \frac{s}{r} \quad (2)$$

Lastly, we put

$$F = 2 \pi r s \quad (3)$$

We eliminate r and s from the three equations and calculate F . We then obtain the weight of the strut.

$$G = \gamma l F = \gamma l \sqrt[3]{\frac{4 P^2 l^2}{\pi k E}}$$

The strut weights are therefore proportional to $\gamma/E^{2/3}$, the inverse of which is the coefficient of merit of the material, $G_m = E^{2/3}/\gamma$.

Since with round tubes, provided the diameter is not reduced for constructive reasons, it is almost always possible to bring the buckling stress near to the yield point and to avoid bulging, the just-computed coefficient of merit of the material is important for streamlined tubes.

4. Buckling and bulging of profiles with flat surfaces.--

This is actually the most common case, especially since it universally applies more or less accurately to so-called open profiles. In considering the bulging of flat surfaces, we put the buckling stress proportional to $E \frac{s^2}{a^2}$ (in which a denotes a dimension of the profile section) and obtain

$G_m = E^{2/3}/\gamma$ by a method similar to the previous one.

5. Buckling stresses of sheet-metal walls, exposed to shearing and compressive forces, for a given distance between the stiffeners.— In this case the buckling stress is again proportional to $E \frac{s^2}{a^2}$ in which a denotes the distance between the stiffeners. We obtain the coefficient of merit of the material $G_m = E^{1/3}/\gamma$.

Comparison of Steel and Electron with Duralumin

	$\sigma_{c.s}$	E	γ
Duralumin	2700	$0.71 \cdot 10^6$	2.8
Steel	6000	$2.2 \cdot 10^6$	7.8
Electron	1900	$0.45 \cdot 10^6$	1.8

The material coefficients of merit were calculated for these numbers. The ratios of these coefficients and consequently the ratios of the strut weights are given in the following table.

Ratios of the Strut Weights

	Steel to duralumin	Electron to duralumin
1. Compression	1.26	0.91
2. Buckling, geom. similar cross section	1.58	0.81
3. Buckling and bulging, curved surface	1.31	0.87
4. " " " flat surface	1.77	0.77
5. Buckling, sheet-metal wall	1.91	0.75

I do not wish to overestimate the importance of these numbers individually, but collectively they show that "dural" is superior to steel in every instance, especially in case 4 of the flat-faced profile closely corresponding to the reality. In many instances case 1 (compression) is probably the best for comparing steel and dural tubes. Of course the weight of the connecting parts (gussets, welds and fittings) are disregarded in this comparison.

In fabric-covered steel airplanes we always use tubes which are welded to the joints. In pure duralumin airplanes, the statically unfavorable flat profiles are generally used because of their being easier to join. The considerations of this section are not applicable therefore to the mutual comparison of such airplanes. Such comparisons must be made with the aid of diagrams like Figure 3. The arguments of this section apply only to the use of steel struts in all-metal airplanes since, in such airplanes, difficulties in assembling the structural parts militate against the use of steel tubes.

The above arguments are directly applicable only to profiles situated inside the airplane parts. In wing struts and the like, the greater frontal area of dural struts naturally increases the drag and detrimentally affects the efficiency of the airplane. In this respect, steel is generally superior, as well as because of its lower cost, generally

smaller weight, and cheaper fittings.

Mounting of experimental struts.— In testing struts they are often compressed between balls. The results, even with very long struts, do not agree with Euler's theory, the reason for which will be explained farther on.

In tests between knife edges, on the contrary, clear and mathematically applicable relations are obtained. Figure 4 represents the upper end of a strut mounted between knife edges. The radii r_1 and r_2 of the knife-edge bearings must be chosen in correspondence with the Hertzian equations. The length of the knife edges must, as explained farther on, be made so large that these radii can be as different as possible, e.g. $r_2 < \frac{2}{3} r_1$.

If the strut (Fig. 4) yields laterally, as a result of the loading, by the angle φ , the line of direction of the force P intersects the axis of the strut at the point A which is distant by

$$\frac{\Delta l}{2} = \frac{1}{\frac{1}{r_2} - \frac{1}{r_1}}$$

from the contact point of the knife edges. The length

Δl is therefore to be regarded as the buckling length.

If the difference between the radii r_1 and r_2 is too small, it may happen that the angle β is greater than the angle of friction, so that the knife edges slip on each other and the relations are obscured.

In compressing the struts between balls (Fig. 5) the strut lengths might be similarly reduced as between knife edges. In order to obtain, according to Hertz, the necessary strength of the balls, either the diameter of the balls must be made unusually large, or the diameter of each ball and its socket must be nearly equal. Thereby the correction Δl becomes very large (up to 30% of the strut length). Furthermore, even for a small yielding, the angle β becomes so large, that rolling no longer occurs, but only slipping. The relations are then no longer calculable, and the buckling stresses scatter more than in the knife bearing. Agreement with the Euler curves cannot generally be obtained. With long struts the experimental points may lie 100% above the Euler curve, if the latter is based on the length of the strut between the centers of the balls.

The objection might be made to the knife bearing that the strut cannot buckle in every direction. In actual practice, however, the direction of buckling of a strut is generally predetermined. Some struts are located in a sheet-metal covering; some are held in one plane by assembly gussets at their ends; and some are crossed by other struts. Often the principal inertia moments also differ considerably, or the struts are compressed eccentrically. The knife bearing then corresponds to the reality as well or better than the ball bearing.

The Index Value of a Structural Part

Under certain conditions, general statements can be made regarding the index value of a structural part. The simplest case, under this head, is that of an eccentrically compressed strut. In this case, as in an axially compressed strut, the strut length l and the buckling load P are established as external structural conditions. Both these conditions, however, are still equivocal. Still another external condition regarding the nature of the eccentricity must be supplied. The commonest condition in airplane construction (as for almost all struts resting on sheet-metal walls and receiving their loads from these walls) is that the load is applied to a marginal surface of the profile as, for example, to the bottom of a U profile in this position. From considerations quite similar to those in the case of an axially compressed strut it again follows that, with a given profile shape and given material with like index value $\sqrt{P/l}$, the same mean compressive stress is admissible; that this stress diminishes with the best profile form with diminishing index value (i.e., with every given index value); and further, that the best profile always bulges or is stressed beyond the yield point, etc. If the mean compressive stress is plotted against the index value, profiles with very different cross-sectional shapes and of different materials can be directly compared.

Even compound girders, consisting of tension and compression members, gussets, etc., can be given index values, when certain conditions regarding the external dimensions and the nature of the loading are known. In a way similar to the one used for the aerodynamic index value, we must determine, for such members, to what length l (e.g., the span of a wing or the width of a seaplane hull) and to what external load P we wish to apply the index value $\sqrt{P/l}$. The considerations are directly applicable only in comparing two such members for which the loads stand in the same ratio which, for example, is usually the case of the bottom girders of flying boats and often also of wing spars. Furthermore, it must be said, by way of qualification, that this comparison is limited to the participation of the parts stressed to their full strength in at least one loading case. Parts, the thickness of whose walls, for example, is determined by other considerations, cannot be thus compared.

In such a structural part no generally applicable relative stress can be given for the comparison of the weights, since the ratio of the stresses of each pair of corresponding members in the two structural parts is not constant for different index values. As the criterion for the weights, we therefore choose the unit weight $G/P l$ (G denoting the weight of the whole structural part) which corresponds to the value $1/\sigma/\gamma$ in the case of a strut. For all tension

members made of a given material the unit weight is the same at all index values. For all compression members it is also the same for two structural parts at the same index value, but, with diminishing index value, the unit weight increases (in contrast with the stress in the strut, whose inverse value it is), and it decreases with increasing index value. This relation between index value and unit weight therefore serves for comparing the forms of a structural part and, in particular, also furnishes a good basis for weight estimates, when similar parts, in other dimensions and with other loads, have been previously tested.

It should be mentioned further that the index values increase with the Rohrbach method of enlarging, while they remain constant with the Lanchester method. Even in this respect the Rohrbach method of enlargement seems to be superior.

G i r d e r s

Comparison of girder sections.— In most of the girders used in an airplane (wing spars and side walls of the fuselage) the tension and compression flanges constitute perfectly distinct structural elements. The height or thickness of these girders is also chiefly determined by external conditions. The following considerations are therefore directly applicable to such girders, but they apply especially to girders which consist of very few distinct members, so that

no clear separation of tension and compression flanges is possible, and in which the height or thickness is determined largely by the strength.

The mathematical computation of the breaking strength of such a girder by the usual method is not very accurate. In the first place, the admissible bending tension in the tension fibers, which is generally 15-30% above the tensile strength, depends on the shape of the cross section or profile. In the second place, as will be shown farther along, bulges always occur on the compression members, even in the most favorable cross-sectional shapes.

The ratio of the weight of the girders to the weight of the cell is comparatively small. It is chiefly the cover profile and the bottom girders of the part which are dimensioned according to their bending stresses. Since, however, special profile shapes are generally used for these parts, it is advisable to make systematic bending tests with these profiles. A good method for evaluating these tests will be described below.

Since tilting is not generally involved with such girders, the bending strength is independent of the profile length except for very short girders. It is therefore necessary (in contrast with struts) to test only one girder of each profile shape. If the cross-sectional area of this girder is designated by F and the bending moment undergone by the

by the girder in breaking by M , it then follows from the law of similarity regarding the strength of materials that another girder with the same profile shape but, for example, with four times as large a cross-sectional area, can withstand eight times as large a bending moment, and that a girder with nine times as large a cross-sectional shape can withstand a moment 27 times as large, etc. Therefore, in all these girders with geometrically similar cross sections, the quantity $F/M^{2/3}$, which we will call the bending constant C , remains the same.

Another girder of the same profile shape, but of relatively thin material, can withstand a greater bending moment in relation to the cross-sectional area, so long as no bulging occurs. The bending constant of its cross section is therefore smaller. If the thickness of the material is further reduced, bulges finally appear, the bending constant then decreases more slowly and finally increases again for exceptionally thin material. In Figure 6 we have plotted the relative thicknesses (e.g., the values a/s for the different profile shapes) on the abscissas and the bending constants on the ordinates. Now, if the bending moment M is given, we obtain the cross-sectional area

$$F = \frac{F}{M^{2/3}} M^{2/3} = C M^{2/3}$$

of the girder to be dimensioned. Hence the smaller C is,

the smaller the requisite cross-sectional area, however great the moment may be. The most favorable profile has therefore the wall-thickness ratio corresponding to the minimum value of σ .

Since, in making the tests, the poorest profile shapes are excluded for bending experiments but, on the other hand, the minimum σ curve is mostly very flat, we obtain (within these limits), for a given profile shape, a nearly uniform bending constant, namely, the minimum which, in our case, is $\sigma =$ about 0.00465.

Since the dashed curve (Fig. 6) which would correspond to the bending strength of the profile if no bulges should appear, continually falls, bulges will always occur (as above mentioned) even in the most favorable cross section.

If rigidity is also considered important, a profile will be selected (within the chosen limits) with the maximum moment of inertia, and consequently a thin-walled profile (Fig. 6, right). On the contrary, if space is limited, a thick-walled profile is chosen. The weight is practically the same in both cases.

The bending constant is no unknown quantity. Like the index value of a strut, it has a value $(\text{kg cm}^{-2})^{-2/3}$, which represents a power of the value of a stress.

Example of dimensioning.— Given $M = 9000 \text{ kg cm}$ (651 lb. ft.) and hence $M^{2/3} = 435$. If we have decided, for constructive reasons, on the profile shown in Figure 6, we thus obtain, with the bending constant corresponding to this profile, the cross-sectional area $F = C M^{2/3} = 0.00465 \times 435 = 2.02 \text{ cm}^2$ (.313 sq.in.). We can now take from the profile table any profile with this cross-sectional area. We will find, for example:

For $F = 2.01$, profile $a = 60$, $s = 1$

" $F = 2.16$, " $a = 70$, $s = 0.9$.

If rigidity is desired, profile 70×0.9 will be chosen.

In Figure 6, we can also plot the C curves for any other profile shape and then directly compare the profiles as regards their suitability for girders. This naturally depends only on the minimum value of C . If profiles of different materials are to be compared, we then plot the value $F\gamma/M^{2/3}$ instead of $F/M^{2/3}$, or choose, as in Figure 3, for the bending constants C , two different scales corresponding to the specific gravities γ .

Center of shearing.— Originally I intended to speak in detail of this important question in metal-aircraft construction. Dr. Engelmann of Danzig, however, called my attention to the fact that articles had already been published on this subject (C. Weber, Zeitschrift für angewandte Mathematik und Mechanik, 1924, p.334; and 1926, p.85). I will therefore lim-

it my remarks to the essential facts regarding the center of shearing and to a few important applications.

If the shearing stress in a girder lies in a plane of symmetry of the cross section, bending occurs, but no torsion. If, however, the girder has no plane of symmetry, or if the bending stress does not lie in the plane of symmetry (as in Figure 7), torsion will generally occur. It can be shown that there is a center of shearing S for every profile cross section. If the shearing stress passes through this point, no torsion occurs.

For the customary thin-walled profiles in metal airplane construction with constant wall thickness s , the determination of the position of the center of shearing is especially simple, as will be shown in the case of the U profile (Fig. 7) with the aid of a shearing stress Q parallel to the web. The occurrence of a shearing force in a girder produces shearing stresses τ , which, in thin-walled profiles, always run in the direction of the sheet metal. These shearing stresses are thus directed in the web of our profile from below upward, in the lower flange toward the left, and in the upper flange toward the right. According to the elementary laws of bending, τs is proportional at every point to the force exerted by the bending stresses (tensile or compressive) on the portion of the girder cross section which lies above this point (e.g., in Fig. 7 for the point 1 proportional

to the strength of the normal stresses in the hatched portion of the cross section). In Figure 7 the magnitude of the shearing stresses is indicated at every point. In determining the position of the center of shearing, the absolute magnitude does not matter, but only the distribution over the profile cross section. The increase of τ s per unit of length in the circumferential direction (in the case of a profile with constant wall thickness therefore also the increase of the shearing stress itself) is proportional at every point to the bending stress at this point and hence proportional to the distance of this point from the neutral fiber. The curves of the shearing tension are therefore straight lines for the flanges and a parabola for the web.

We will now compute the magnitude of the shearing stress in the different cross sections. With a constant τ_0 (which subsequently disappears again) we obtain

$$Q_b = \frac{1}{2} \tau_0 b s$$

for the upper and lower flanges and

$$Q_s = \left(\tau_0 + \frac{2}{3} \tau_0 \frac{a}{b} \right) a s$$

for the web. The resultant shearing stress Q , which has the magnitude Q_s , therefore, lies at the distance

$$q = \frac{a Q_b}{Q_s} = \frac{b}{2 + \frac{1}{3} \frac{a}{b}}$$

from the profile web. For $a = 3b$ we obtain, e.g.,
 $q = 0.33b$.

In our case the resultant shearing force therefore lies outside the profile, and its point of intersection with the axis of symmetry of the cross section determines the center of shearing S . If the external force Q also passes through this point, its equilibrium can then be maintained by the shearing stresses in the profile cross section without the development of torsion. Otherwise, torsion is produced which may be very great in open thin-walled profiles and must be prevented by constructive measures.

Moreover, it can be easily shown that the center of shearing coincides with the point of the profile cross section, which does not change its position in a simple torsion of the profile.

In an angular profile the center of shearing is at the apex of the profile. In a girder (Fig. 8) consisting of two very strong spars and a very thin curved sheet for absorbing the shearing forces (Such girders often occur in metal airplane construction) the center of shearing (when the participation of said sheet in the absorption of the bending moment is disregarded) is $q = \frac{b}{h} r = \frac{\varphi}{\sin\varphi} r$ distant from the center of curvature. The constant magnitude of the shearing stress is $\tau = \frac{Q}{h s}$.

The example of the float cross section (Fig. 9) shows how the center of shearing is calculated for closed profiles. The spars are supposed to absorb the whole bending moment, the effect of the longitudinal stresses in the thin metal sheet being disregarded. It is assumed that the width of the spar flanges is small in comparison with the developed length of the sheet. If we calculate the mutual displacement of the side spars in the longitudinal direction in the case of simple lateral bending due to a shearing force Q (the top and bottom spars having no effect on these phenomena), in the first place from the deformation of the lower sheet and in the second place from the deformation of the upper sheet, we must obtain the same result in both cases. The following formula (in which G = modulus of shear) must then apply.

$$\frac{\tau_1 r \pi}{G} = \frac{\tau_2 2 \sqrt{h^2 + r^2}}{G}$$

The resultant shearing stress in the upper sheet is therefore (Fig. 8)

$$Q_1 = \tau_1 2 r s_1$$

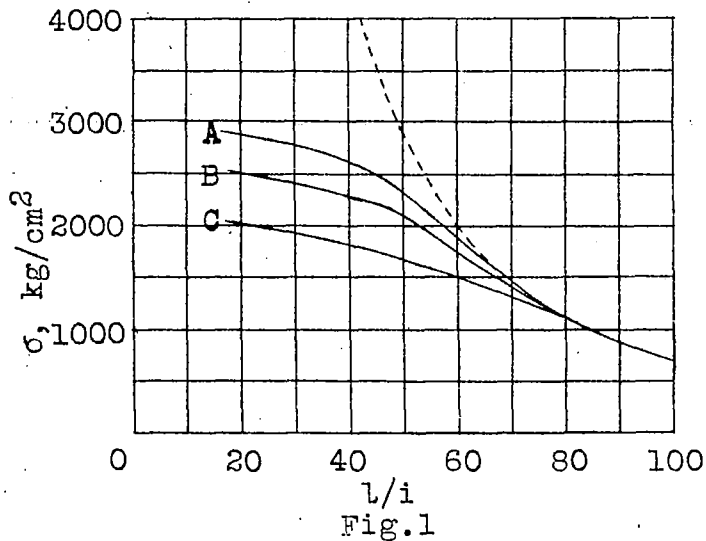
at the distance $h + \frac{\pi}{2} r$ from 0, and the resultant shearing stress in the lower sheet is

$$Q_2 = \tau_2 2 r s_2 = \tau_1 \frac{r \pi}{2 \sqrt{h^2 + r^2}} 2 r s_2$$

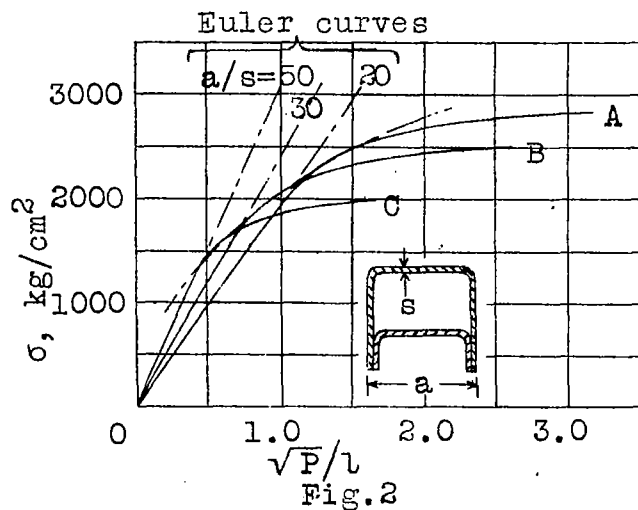
at 0. Hence the position of the center of shearing S is

$$q = \frac{Q_1 \left(h + \frac{\pi}{2} r \right)}{Q_1 + Q_2} = \frac{h + \frac{\pi}{2} r}{1 + \frac{S_2}{S_1} \frac{r \pi}{2 \sqrt{h^2 + r^2}}}.$$

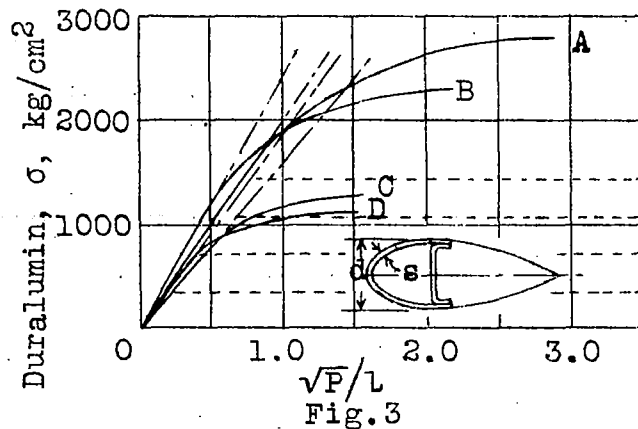
Translation by Dwight M. Miner,
National Advisory Committee
for Aeronautics.



A, $a/s = 20$
 B, $a/s = 30$
 C, $a/s = 50$



A, $a/s = 20. a = 2.21\sqrt{F}$
 B, $a/s = 30. a = 2.68\sqrt{F}$
 C, $a/s = 50. a = 3.40\sqrt{F}$



A, $s/d = .04$ } Duralumin with
 B, $s/d = .03$ } $\sigma_{0.2} = 27 \text{ kg/mm}^2$
 $\sigma_B = 38$ "

C, $s/d = .03$ } Steel with
 D, $s/d = .02$ } $\sigma_{0.2} = 38 \text{ kg/mm}^2$
 $\sigma_B = 50$ "

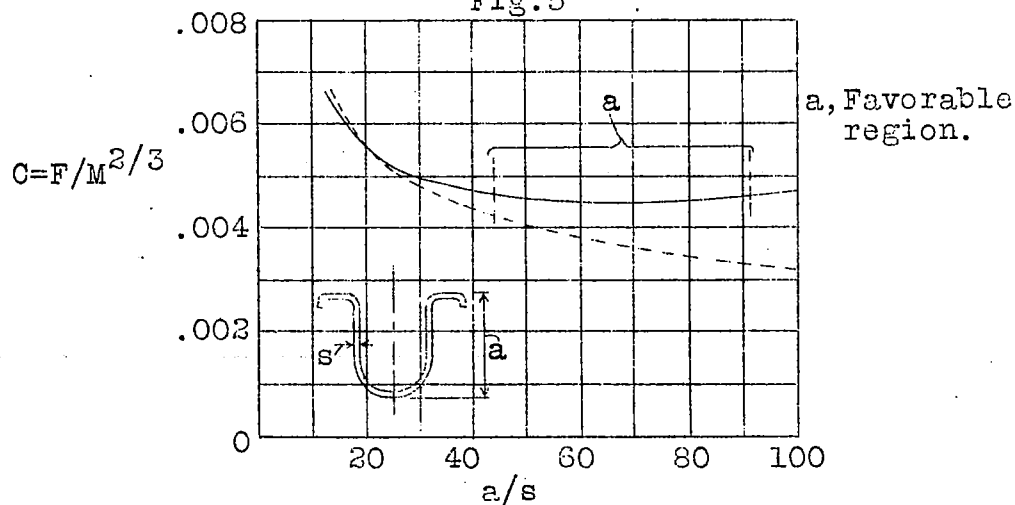
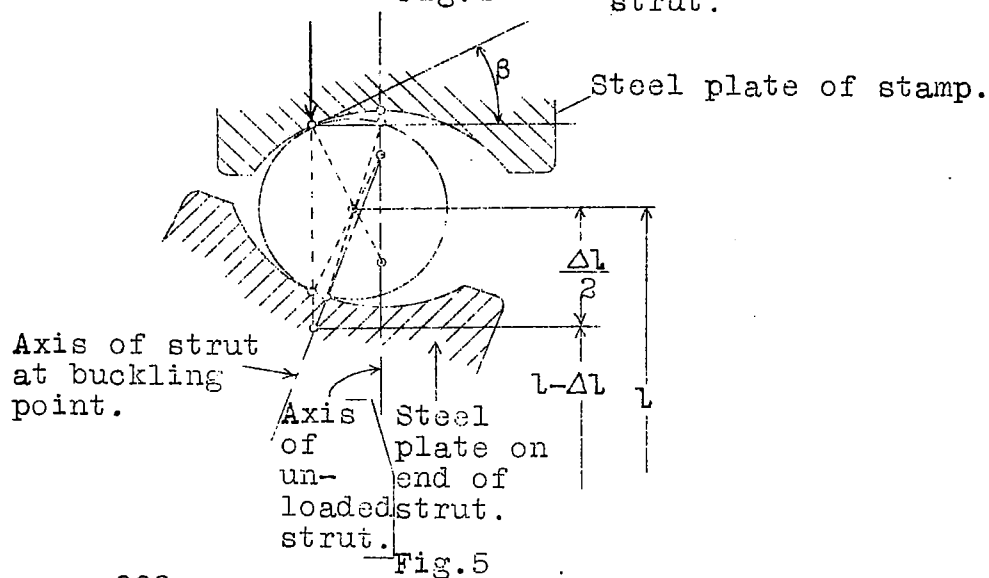
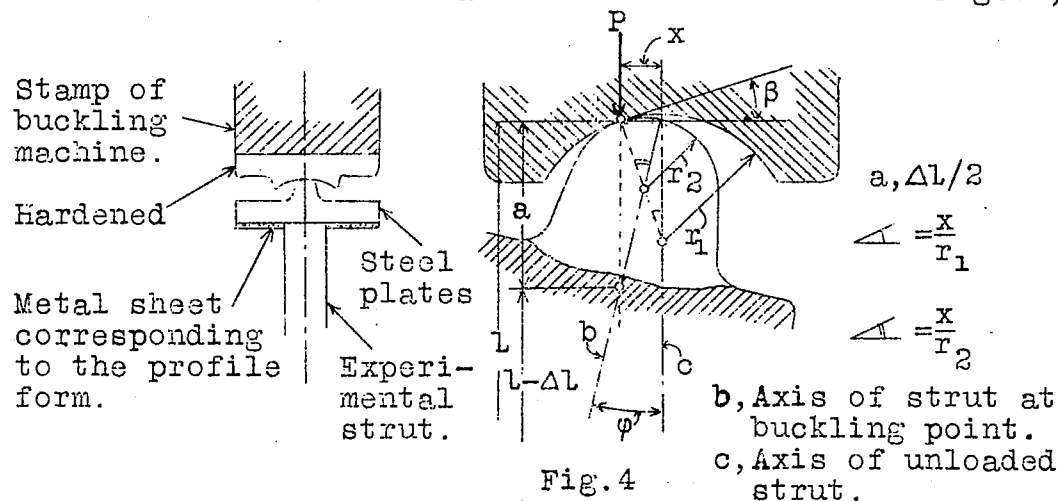


Fig. 6

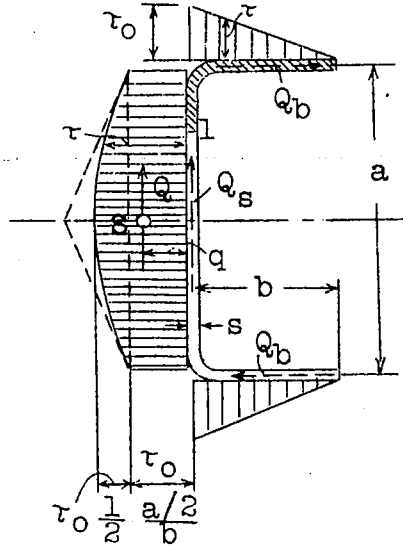


Fig. 7

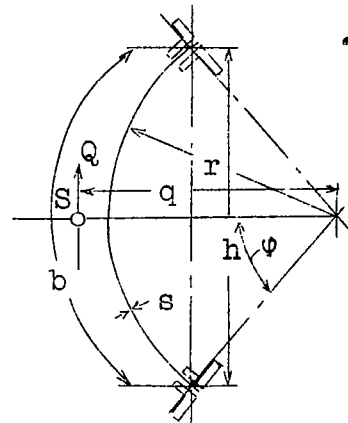


Fig. 8

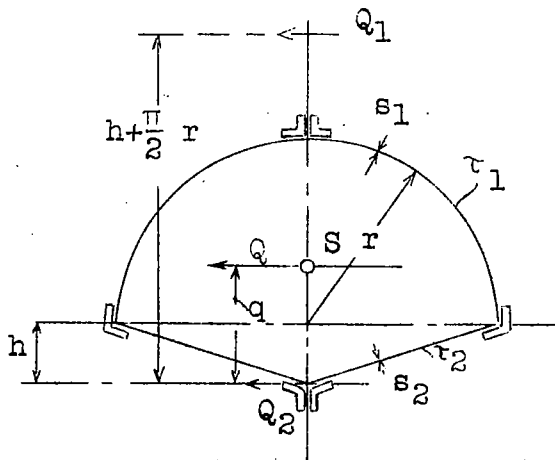


Fig.9

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